Model Solutions

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

Friday 22 May 2020

Afternoon (Time: 1 hour 30 minutes) | Paper Reference **9FM0/3A**

Further Mathematics

Advanced

Paper 3A: Further Pure Mathematics 1

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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Pearson

1. Use l'Hospital's Rule to show that

$$\lim_{x \to \frac{\pi}{2}} \frac{(e^{\sin x} - \cos(3x) - e)}{\tan(2x)} = -\frac{3}{2}$$

(5)

$$\frac{\frac{d}{dx}\left(e^{\sin x} - \cos(3x) - e\right)}{\frac{d}{dx}\left(\tan(2x)\right)}$$

$$= \frac{\cos x e^{\sin x} + 3\sin(3x)}{2\sec^2(2x)}$$

$$\Rightarrow \lim_{x \to \frac{\pi}{2}} \frac{\cos x e^{\sin x} + 3\sin(3x)}{2\sec^2(2x)}$$

$$= \frac{\cos(\frac{\pi}{2})e^{\sin(\frac{\pi}{2})} + 3\sin\left(\frac{3\pi}{2}\right)}{2\sec^2\left(\frac{2\pi}{2}\right)}$$

$$= \frac{0 \times e^1 + 3(-1)}{2(-1)^2} = \frac{-3}{2}$$ (as required)

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Question 1 continued

(Total for Question 1 is 5 marks)

2.

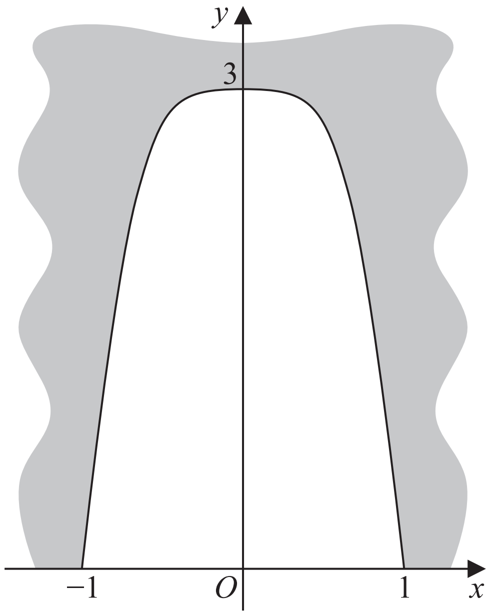

Figure 1

Figure 1 shows a sketch of the vertical cross-section of the entrance to a tunnel. The width at the base of the tunnel entrance is 2 metres and its maximum height is 3 metres.

The shape of the cross-section can be modelled by the curve with equation $y = f(x)$ where

$$f(x) = 3\cos\left(\frac{\pi}{2}x^2\right) \qquad x \in [-1, 1]$$

A wooden door of uniform thickness 85 mm is to be made to seal the tunnel entrance.

Use Simpson's rule with 6 intervals to estimate the volume of wood required for this door, giving your answer in m³ to 4 significant figures.

(6)

	y_0	y_1	y_2	y_3	y_4	y_5	y_6
x	-1	$-2/3$	$-1/3$	0	$1/3$	$2/3$	1
y	0	2.2981	2.9544	3	2.9544	2.2981	0

$y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + 4y_5 + y_6$
$= 0 + (4\times 2.2981) + (2\times 2.9544) + (4\times 3) + (2\times 2.9544) +$
$(4\times 2.2981) + 0$
$= 42.203$

$\therefore$ volume $\simeq \frac{85}{1000} \times \frac{\frac{1}{3}}{3} \times 42.203 \simeq 0.3986\ m^3$
(4sf)

P 6 2 6 7 3 A 0 4 2 8

Question 2 continued

(Total for Question 2 is 6 marks)

3. The points A, B and C, with position vectors $\mathbf{a} = 3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$, $\mathbf{b} = \mathbf{i} + 4\mathbf{j} + 5\mathbf{k}$ and $\mathbf{c} = -2\mathbf{i} + 3\mathbf{j} + 3\mathbf{k}$ respectively, lie on the plane Π

(a) Find $\overrightarrow{AB} \times \overrightarrow{AC}$ **(3)**

(b) Find an equation for Π in the form $\mathbf{r}.\mathbf{n} = p$ **(2)**

The point D has position vector $8\mathbf{i} + 7\mathbf{j} + 5\mathbf{k}$

(c) Determine the volume of the tetrahedron $ABCD$ **(4)**

a) $\vec{AB} = \begin{pmatrix} 1 \\ 4 \\ 5 \end{pmatrix} - \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} = \begin{pmatrix} -2 \\ 6 \\ 4 \end{pmatrix}$

$\vec{AC} = \begin{pmatrix} -2 \\ 3 \\ 3 \end{pmatrix} - \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} = \begin{pmatrix} -5 \\ 5 \\ 2 \end{pmatrix}$

$\vec{AB} \times \vec{AC}$ is the cross product :

$$\begin{vmatrix} i & j & k \\ -2 & 6 & 4 \\ -5 & 5 & 2 \end{vmatrix} = \begin{pmatrix} (6\times2) - (5\times4) \\ -((2\times-2) - (-5\times4)) \\ (-2\times5) - (-5\times6) \end{pmatrix}$$

$$= \boxed{\begin{pmatrix} -8 \\ -16 \\ 20 \end{pmatrix}}$$

b) $n = \begin{pmatrix} -8 \\ -16 \\ 20 \end{pmatrix}$ let $r = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$ *could be any of the given points*

$r \cdot n = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -8 \\ -16 \\ 20 \end{pmatrix} = (3\times-8) + (-2\times-16) + (1\times20)$

$= 28$

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Question 3 continued

$\therefore$ equation for Π :

$$\boxed{r \cdot \begin{pmatrix} -8 \\ -16 \\ 20 \end{pmatrix} = 28}$$

c) $\vec{AD} \cdot (\vec{AB} \times \vec{AC})$

$$\vec{AD} = \begin{pmatrix} 8 \\ 7 \\ 5 \end{pmatrix} - \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \\ 9 \\ 4 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 5 \\ 9 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} -8 \\ -16 \\ 20 \end{pmatrix} = (-8 \times 5) + (9 \times -16) + (4 \times 20)$$

$$= -104$$

$$\text{Volume} = \frac{1}{6} |-104| = \boxed{\frac{52}{3} \text{ units}^3}$$

Question 3 continued

Question 3 continued

(Total for Question 3 is 9 marks)

4.

$$f(x) = x^4 \sin(2x)$$

Use Leibnitz's theorem to show that the coefficient of $(x - \pi)^8$ in the Taylor series expansion of $f(x)$ about π is

$$\frac{a\pi + b\pi^3}{315}$$

where a and b are integers to be determined.

(8)

The Taylor series expansion of $f(x)$ *about* $x = k$ *is given by*

$$f(x) = f(k) + (x-k)f'(k) + \frac{(x-k)^2}{2!}f''(k) + \ldots + \frac{(x-k)^r}{r!}f^{(r)}(k) + \ldots$$

$f(x) = x^4 \sin(2x)$

let $u = x^4$

$u' = 4x^3$

$u'' = 12x^2$

$u''' = 24x$

$u'''' = 24$

$u^n = 0$ (for $n > 4$)

let $v = \sin(2x)$

$v' = 2\cos(2x)$

$v'' = -4\sin(2x)$

$v''' = -8\cos(2x)$

$v^{IV} = 16\sin(2x)$

$v^{V} = 32\cos(2x)$

$v^{VI} = -64\sin(2x)$

$v^{VII} = -128\cos(2x)$

$v^{VIII} = 256\sin(2x)$

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Question 4 continued

Using Leibnitz's Theorem:

$$f^8(x) = x^4 \times 256\sin(2x) + (8 \times 4x^3 \times -128\cos(2x))$$
$$+\left(\left(\frac{8\times 7}{2!}\right) \times 12x^2 \times -64\sin(2x)\right) + \left(\left(\frac{8\times7\times6}{3!}\right)\times 24x \times 32\cos(2x)\right)$$
$$+\left(\frac{8\times7\times6\times5}{4!} \times 24 \times 16\sin(2x)\right) + 0 + \ldots$$

when $x = \pi$, all $\sin(2x)$ terms $= 0$ and all $\cos(2x)$ terms $= 1$:

$$f^8(\pi) = (8 \times 4\pi^3 \times -128) + \left(\frac{8\times7\times6}{3!} \times 24\pi \times 32\right)$$
$$= -4096\pi^3 + 43008\pi$$

Coefficient is $\frac{f^8(\pi)}{8!} = \frac{-4096\pi^3 + 43008\pi}{8!}$

$$= \frac{336\pi - 32\pi^3}{315}$$

$a = 336 \quad b = -32$

Question 4 continued

Question 4 continued

(Total for Question 4 is 8 marks)

5. The ellipse E has equation

$$\frac{x^2}{36} + \frac{y^2}{16} = 1$$

The points S and S' are the foci of E.

(a) Find the coordinates of S and S'

(3)

(b) Show that for any point P on E, the triangle PSS' has constant perimeter and determine its value.

(4)

a) $b^2 = a^2(1-e^2)$

$16 = 36(1-e^2)$

$16 = 36 - 36e^2$

$36e^2 = 20$

$e^2 = \frac{20}{36}$

$e^2 = \frac{5}{9}$

$e = \frac{\sqrt{5}}{3}$

S and S' $\Rightarrow$ Foci are $(\pm 2\sqrt{5}, 0)$

b) Perimeter $= PS + PS' + SS'$

where $PS + PS' = e(PM + PM')$

$= e \times \frac{2a}{e}$

$PS + PS' + SS' = 2 \times 2\sqrt{5} + 12$

$= 12 + 4\sqrt{5}$

Hence perimeter is constant for any P and E

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Question 5 continued

(Total for Question 5 is 7 marks)

6. A physics student is studying the movement of particles in an electric field. In one experiment, the distances in micrometres of two moving particles, A and B, from a fixed point O are modelled by

$$d_A = |5t - 31|$$
$$d_B = |3t^2 - 25t + 8|$$

respectively, where t is the time in seconds after motion begins.

(a) Use algebra to find the range of time for which particle A is further away from O than particle B is from O.

(8)

It was recorded that the distance of particle B from O was less than the distance of particle A from O for approximately 4 seconds.

(b) Use this information to assess the validity of the model.

(2)

a) $|5t-31| > |3t^2-25t+8|$

C.V. $5t-31 = 3t^2-25t+8$

$0 = 3t^2 - 30t + 39$ (divide both sides by 3)

$0 = t^2 - 10t + 13$

completing the square:

$0 = (t-5)^2 - 25 + 13$

$0 = (t-5)^2 - 12$

$12 = (t-5)^2$

$\pm 2\sqrt{3} = t-5$

$5 \pm 2\sqrt{3} = t$

CV. $-(5t-31) = 3t^2 - 25t + 8$

$31 - 5t = 3t^2 - 25t + 8$

$0 = 3t^2 - 20t - 23$

$t = \frac{23}{3}$ $\quad t = -1$

Both regions:

$-1 < t < 5 - 2\sqrt{3}$

$\frac{23}{3} < t < 5 + 2\sqrt{3}$

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Question 6 continued

b) Time that B is closer to O than particle A is

$5+2\sqrt{3} - \frac{23}{3} + 5 - 2\sqrt{3} = \frac{7}{3}$ seconds.

This is less than 4 seconds so the model does not seem appropriate.

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Turn over

Question 6 continued

Question 6 continued

(Total for Question 6 is 10 marks)

7. The points $P(9p^2, 18p)$ and $Q(9q^2, 18q)$, $p \neq q$, lie on the parabola C with equation

$$y^2 = 36x$$

The line l passes through the points P and Q

(a) Show that an equation for the line l is

$$(p + q)y = 2(x + 9pq)$$

(3)

The normal to C at P and the normal to C at Q meet at the point A.

(b) Show that the coordinates of A are

$$\left(9(p^2 + q^2 + pq + 2), -9pq(p + q)\right)$$

(7)

Given that the points P and Q vary such that l always passes through the point (12, 0)

(c) find, in the form $y^2 = \mathrm{f}(x)$, an equation for the locus of A, giving $\mathrm{f}(x)$ in simplest form.

(4)

Gradient of PQ

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{18p - 18q}{9p^2 - 9q^2} = \frac{18(p-q)}{9(p^2 - q^2)} = \frac{2\cancel{(p-q)}}{(p+q)\cancel{(p-q)}}$$

$$= \frac{2}{p+q}$$

Using $y = mx + c$

$$18p = \left(\frac{2}{p+q}\right) 9p^2 + c$$

$$18p = \frac{18p^2}{p+q} + c$$

$$\frac{18p(p+q)}{p+q} = \frac{18p^2}{p+q} + c$$

$$\frac{18p^2 + 18pq - 18p^2}{p+q} = c$$

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Question 7 continued

$\frac{18pq}{p+q} = c$

$\therefore y = \frac{2x}{p+q} + \frac{18pq}{p+q}$

$y(p+q) = 2x + 18pq$

$\therefore (p+q)y = 2(x + 9pq)$ (as required)

b) $y^2 = 36x$

Differentiating both sides:

$2y \frac{dy}{dx} = 36$

$\frac{dy}{dx} = \frac{36}{2y}$

At point P, $y = 18p$

$\frac{dy}{dx} = \frac{36}{2(18p)} = \frac{1}{p}$ $\therefore$ gradient of tangent at P $= \frac{1}{p}$

So gradient normal to P $= -p$

At point Q, $y = 18q$

$\frac{dy}{dx} = \frac{36}{2(18q)} = \frac{1}{q}$

$\therefore$ gradient of tangent at Q $= \frac{1}{q}$ so gradient normal to Q $= -q$

Question 7 continued

Using $y - y_1 = m(x - x_1)$

Line normal to P:

$y - 18p = -p(x - 9p^2)$

$y - 18p = -px + 9p^3$

$y = -px + 9p^3 + 18p$

Line normal to Q:

$y - 18q = -q(x - 9q^2)$

$y = -qx + 9q^3 + 18q$

Equalling both lines to eachother:

$-qx + 9q^3 + 18q = -px + 9p^3 + 18p$

$-qx + px = 9p^3 + 18p - 9q^3 - 18q$

$x(p - q) = 9(p^3 - q^3) + 18(p - q)$

$x = \dfrac{9(p^3 - q^3) + 18(p - q)}{p - q}$

$x = \dfrac{9(p-q)((p^2 + pq + q^2) + 2)}{p - q}$

$x = 9(p^2 + pq + q^2 + 2)$

Sub x into y:

$y = -q(9(p^2 + pq + q^2 + 2)) + 9q^3 + 18q$

$= -9q(p^2 + pq)$

$y = -9pq(p + q)$

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Question 7 continued

Coordinates A: $\left(9(p^2+q^2+pq+2), -9pq(p+q)\right)$

(as required)

c) sub $x=12$, $y=0$ into line l:

$(p+q)\times 0 = 2(12 + 9pq)$

$0 = 24 + 18pq$

$-24 = 18pq$

$-\frac{4}{3} = pq$

Sub pq value into A coordinates:

$9\left(p^2+q^2-\frac{4}{3}+2\right) = x$

$9p^2+9q^2+6 = x$

$-9\left(-\frac{4}{3}\right)(p+q) = y$

$12(p+q) = y$

$\therefore\ y^2 = 12^2(p+q)^2$

$= 144(p^2+2pq+q^2)$

$= 144\left(\frac{x}{9} + 2\left(-\frac{4}{3}\right) - \frac{2}{3}\right)$

$\Rightarrow\ y^2 = 16(x-30)$

$y^2 = 16x - 480$

(Total for Question 7 is 14 marks)

8.
$$f(x) = \frac{3}{13 + 6\sin x - 5\cos x}$$

Using the substitution $t = \tan\left(\frac{x}{2}\right)$

(a) show that $f(x)$ can be written in the form

$$\frac{3(1+t^2)}{2(3t+1)^2 + 6}$$

(3)

(b) Hence solve, for $0 < x < 2\pi$, the equation

$$f(x) = \frac{3}{7}$$

giving your answers to 2 decimal places where appropriate.

(5)

(c) Use the result of part (a) to show that

$$\int_{\frac{\pi}{3}}^{\frac{4\pi}{3}} f(x)\,dx = K\left(\arctan\left(\frac{\sqrt{3}-9}{3}\right) - \arctan\left(\frac{\sqrt{3}+3}{3}\right) + \pi\right)$$

where K is a constant to be determined.

(8)

a) let $t = \tan\left(\frac{x}{2}\right)$

As we know $\tan\left(\frac{x}{2}\right) = \frac{1-\cos x}{\sin x}$

$\therefore \tan^2\left(\frac{x}{2}\right) = \frac{(1-\cos x)^2}{\sin^2 x} = \frac{1-2\cos x+\cos^2 x}{\sin^2 x}$

$1+t^2 = \frac{\sin^2 x}{\sin^2 x} + \frac{1-2\cos x+\cos^2 x}{\sin^2 x}$

$= \frac{(\sin^2 x+\cos^2 x)+1-2\cos x}{\sin^2 x}$

$= \frac{2(1-\cos x)}{1-\cos^2 x} = \frac{2(1-\cos x)}{(1-\cos x)(1+\cos x)} = \frac{2}{1+\cos x}$

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Question 8 continued

$$f(x) = \frac{3}{13 + 6\sin x - 5\cos x}$$

Find $\sin x$ in terms of t:

$$2\left(\frac{1-\cos x}{\sin x}\right) \times \left(\frac{1+\cos x}{2}\right) = \frac{2t}{1+t^2}$$

$$\frac{1-\cos^2 x}{\sin x} = \frac{\sin^2 x}{\sin x} = \sin x = \frac{2t}{1+t^2}$$

find $\cos x$ in terms of t:

$$\frac{1-t^2}{1+t^2} = \cos x$$

$$\therefore f(x) = \frac{3}{13 + 6\left(\frac{2t}{1+t^2}\right) - 5\left(\frac{1-t^2}{1+t^2}\right)}$$

Multiply denominator and numerator by $1+t^2$

$$f(x) = \frac{3(1+t^2)}{13(1+t^2) + 12t - 5(1-t^2)}$$

$$f(x) = \frac{3(1+t^2)}{13 + 13t^2 + 12t - 5 + 5t^2}$$

$$= \frac{3(1+t^2)}{18t^2 + 12t + 8} = \frac{3(1+t^2)}{2(9t^2 + 6t + 1) + 6}$$

Question 8 continued

$$= \frac{3(1+t^2)}{2[(3t+1)^2-1]+8}$$

$$= \frac{3(1+t^2)}{2(3t+1)^2+6} \quad \text{(as required)}$$

b) If $f(x) = \frac{3}{7}$

$$\Rightarrow \frac{3(1+t^2)}{2(3t+1)^2+6} = \frac{3}{7}$$

$$7(1+t^2) = 2(3t+1)^2+6$$
$$7+7t^2 = 2(9t^2+6t+1)+6$$
$$7+7t^2 = 18t^2+12t+8$$
$$0 = 11t^2+12t+1$$
$$0 = (11t+1)(t+1)$$

$$t = -1, -\frac{1}{11}$$

when $t = -1$

$$x = 2\arctan(-1) + 2\pi$$
$$x = \frac{3\pi}{2}$$

when $t = -\frac{1}{11}$

$$x = 2\arctan\left(-\frac{1}{11}\right) + 2\pi$$
$$= 6.10 \text{ (3sf)}$$

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Question 8 continued

c) $\int f(x)\ dx$

Limits

when $x = \frac{4\pi}{3}$

As $t = \tan\left(\frac{x}{2}\right)$

$t = -\sqrt{3}$

$2\arctan(t) = x$

when $x = \frac{\pi}{3}$

$\frac{dx}{dt} = \frac{2}{1+t^2}$

$t = \frac{\sqrt{3}}{3}$

$\therefore dx = \frac{2}{1+t^2}\ dt$

$\therefore \int_{\frac{\pi}{3}}^{\frac{4\pi}{3}} f(x)\,dx = \int_{\frac{\pi}{3}}^{\pi} f(x)\ dx + \int_{\pi}^{\frac{4\pi}{3}} f(x)\ dx$

$\lim_{a\to\infty} \int_{\frac{\pi}{3}}^{a} f(x)\ dx + \int_{a}^{\frac{4\pi}{3}} f(x)\ dx$

$\Rightarrow \lim_{a\to\infty} \int_{\frac{\sqrt{3}}{3}}^{a} \frac{3(1+t^2)}{2(3t+1)^2+6} \times \frac{2}{1+t^2}\ dt$

$+ \lim_{a\to\infty} \int_{-a}^{-\sqrt{3}} \frac{3(1+t^2)}{2(3t+1)^2+6} \times \frac{2}{1+t^2}\ dt$

$= \lim_{a\to\infty} \int_{\frac{\sqrt{3}}{3}}^{a} \frac{3}{(3t+1)^2+3}\ dt + \lim_{a\to\infty} \int_{-a}^{-\sqrt{3}} \frac{3}{(3t+1)^2+3}\ dt$

Question 8 continued

Using substitution

Let $u = 3t+1$

$\frac{du}{dt} = 3$

$dt = \frac{1}{3} du$

Limits

when $t = -\sqrt{3}$

$u = -3\sqrt{3}+1$

when $t = \frac{\sqrt{3}}{3}$

$u = \sqrt{3}+1$

$$\lim_{a\to\infty} \int_a^{-3\sqrt{3}+1} \frac{3}{u^2+3} \times \frac{1}{3}\, du + \lim_{a\to\infty} \int_{-a}^{\sqrt{3}+1} \frac{3}{u^2+3} \times \frac{1}{3}\, du$$

$$= \lim_{a\to\infty} \int_a^{-3\sqrt{3}+1} \frac{1}{u^2+3}\, du + \lim_{a\to\infty} \int_{-a}^{\sqrt{3}+1} \frac{1}{u^2+3}\, du$$

$$= \left[\frac{1}{\sqrt{3}} \arctan\left(\frac{u}{\sqrt{3}}\right)\right]_{\sqrt{3}+1}^{-3\sqrt{3}+1} + \ldots$$

$$= \frac{1}{\sqrt{3}} \arctan\left(\frac{-3\sqrt{3}+1}{\sqrt{3}}\right) - \frac{1}{\sqrt{3}} \arctan\left(\frac{\sqrt{3}+1}{\sqrt{3}}\right) + \ldots$$

$$= \frac{\sqrt{3}}{3} \arctan\left(\frac{\sqrt{3}-9}{3}\right) - \frac{\sqrt{3}}{3} \arctan\left(\frac{3+\sqrt{3}}{3}\right) + \ldots$$

When $a \to \infty$ $\quad \frac{\pi}{2\sqrt{3}} + \frac{\pi}{2\sqrt{3}} \to \frac{\pi}{\sqrt{3}}$

$$\boxed{\therefore \frac{\sqrt{3}}{3}\left(\arctan\left(\frac{\sqrt{3}-9}{3}\right) - \arctan\left(\frac{3+\sqrt{3}}{3}\right) + \pi\right)}$$

(Total for Question 8 is 16 marks)

TOTAL FOR PAPER IS 75 MARKS